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The influence of recycle on double-pass heat and mass transfer through a parallel-plate device

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Abstract

The influence of recycle on double-pass parallel-plate heat and mass exchangers with uniform wall temperature has been studied analytically. The heat or mass transfer problem is solved for fully developed laminar velocity profiles in parallel-plate channels by ignoring axial conduction, and with fluid properties of temperature independence. Analytical results show that recycle can effectively enhance the heat or mass transfer rate compared with that in an open conduit (without an impermeable plate barrier inserted for double-pass and recycle operation). The desirable preheating effect and the undesirable effect of decreasing residence time are the two conflict effects produced by the recycle operation. It was found that the increase in preheating effect by increasing the reflux ratio can generally compensate for the decrease of residence time, leading to improved performance, exceptionally for very small Graetz numbers. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

B conduit width [m]
c_p specific heat at constant pressure [kJ/kg·K]
D_e equivalent diameter of conduit [m]
d_{mn} coefficient in the eigenfunction $F_{a,m}$
e_{mn} coefficient in the eigenfunction $F_{b,m}$
F_m eigenfunction associated with eigenvalue λ_m
f friction factor
G_Z Graetz number, $W/\alpha BL$
G_m function defined during the use of orthogonal expansion method
$\bar{h}$ average heat transfer coefficient [kW/m^2·K]
I_h improvement of heat transfer, defined by equation (58)
I_p increment of power consumption, defined by equation (65)
k thermal conductivity of the fluid [kW/m·K]
L conduit length [m]
ℓw_f friction loss in conduit, [kW·s/kg]
$\overline{Nu}$ Nusselt number
R reflux ratio, reverse volume flow rate divided by input volume flow rate
Re Reynolds number
S_m expansion coefficient associated with eigenvalue λ_m
T temperature of fluid [K]
T_I inlet temperature of fluid in conduit [K]
T_s wall temperature [K]
V input volume flow rate of conduit [m^3/s]
v velocity distribution of fluid [m/s]
$\bar{v}$ average velocity of fluid [m/s]
W distance between two parallel plates [m]
x transversal coordinate [m]
z longitudinal coordinate [m].

Greek symbols

α thermal diffusivity of fluid [m^2/s]
Δ ratio of channel thickness, W_a/W
η transversal coordinate, x/W
θ dimensionless temperature $(T-T_I)/(T_s-T_I)$
λ_m eigenvalue
ξ longitudinal coordinate, z/L
ρ density of the fluid [kg/m^3]
ψ dimensionless temperature $(T-T_s)/(T_I-T_s)$.

Subscripts

a in forward flow channel
b in backward flow channel
F at the outlet of a double-pass device

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I at the inlet
L at the outlet, $\xi = 1$
0 in a single-pass device without recycle
s at the wall surface.

1. Introduction

The early studies of laminar heat and mass transfer in a bounded conduit is known as Graetz problems [1, 2] with ignoring axial conduction or diffusion. The extension of Graetz problems to involve axial conduction or diffusion, particularly for low Prandtl number fluids such as liquid metals, is called extended Graetz problems [3–10]. Recently, numerous notable papers have been studied to treat the conjugated conduction–convection problem, as in conjugated Graetz problems [11–17], and brought up to deal with heat and/or mass transfer processes when fully energy equations for two or more contiguous phases or streams are to be solved simultaneously.

Loop reactors [18, 19], air-lift reactors [20, 21] and draft-tube bubble columns [22, 23], which have been developed with internal or external refluxes at both ends, are well-known devices in separation processes and reactor designs, and are widely used in absorption, fermentation, and polymerization. The reflux ratio and the position of an impermeable plate barrier, which is inserted for recycle operation, are the two important factors to be considered in designing heat and mass transfer processes with internal and external refluxes at both ends. For this purpose the fully developed laminar velocity profile is assumed and the conjugated Graetz problem is discussed for uniform wall temperature. The analytical solutions of Graetz problems or conjugated Graetz problems were essentially performed by the use of orthogonal expansion techniques [24–32], and a numerical and experimental study was carried out in the previous work [33].

It is the purpose of this study to develop the complete theory and investigate the improvement of performance in double-pass parallel-plate heat and mass exchangers with external recycle at the outlet of such devices.

2. Temperature distribution in a double-pass device with recycle

Consider the heat transfer in two channels with thickness ΔW and $(1-\Delta)W$, respectively, which is obtained by inserting an impermeable plate barrier with negligible thickness and thermal resistance, into a parallel conduit of thickness W, length L, and width B ($\gg W$), as shown in Fig. 1. Before entering the upper channel, the fluid with volume flow rate V and the outlet temperature T_L from the lower channel, will mix with the recycle fluid of volume flow rate RV and outlet temperature T_F, which is controlled by means of a conventional pump situated at the end of upper channel.

After the following assumptions are made: constant physical properties and wall temperatures; purely fully-developed laminar flow in each channel; negligible end effects and axial conduction or diffusion; the velocity distributions and equations of energy in dimensionless form may be obtained as

$$\frac{\partial^2 \psi_a(\eta_a,\xi)}{\partial \eta_a^2} = \left(\frac{W_a^2 v_a}{L\alpha}\right)\frac{\partial \psi_a(\eta_a,\xi)}{\partial \xi} \tag{1}$$

$$\frac{\partial^2 \psi_b(\eta_b,\xi)}{\partial \eta_b^2} = \left(\frac{W_b^2 v_b}{L\alpha}\right)\frac{\partial \psi_b(\eta_b,\xi)}{\partial \xi} \tag{2}$$

$$v_a(\eta_a) = \overline{v_a}(6\eta_a - 6\eta_a^2), \quad 0 \leqslant \eta_a \leqslant 1 \tag{3}$$

$$v_b(\eta_b) = \overline{v_a}(6\eta_b - 6\eta_b^2), \quad 0 \leqslant \eta_b \leqslant 1 \tag{4}$$

in which

$$\overline{v_a} = [V/W_a B], \quad \overline{v_b} = -[V(R+1)/W_b B],$$

$$\eta_a = \frac{x_a}{W_a}\eta_b = \frac{x_b}{W_b}, \quad \xi = \frac{z}{L}$$

$$G_Z = \frac{V(W_a+W_b)}{\alpha BL} = \frac{VW}{\alpha BL}, \quad W_a = \Delta W,$$

$$W_b = (1-\Delta)W, \quad \frac{W_a}{W_b} = \frac{\Delta}{1-\Delta}$$

$$\psi_a = \frac{T_a - T_s}{T_I - T_s}, \quad \psi_b = \frac{T_b - T_s}{T_I - T_s},$$

$$\theta_a = 1-\psi_a = \frac{T_a - T_I}{T_s - T_I}, \quad \theta_b = 1-\psi_b = \frac{T_b - T_I}{T_s - T_I}. \tag{5}$$

The boundary conditions for solving equations (1) and (2) are

$$\psi_a(0,\xi) = 0 \tag{6}$$

$$\psi_b(0,\xi) = 0 \tag{7}$$

$$\psi_a(1,\xi) = \psi_b(1,\xi) \tag{8}$$

$$-\frac{\partial \psi_a(1,\xi)}{\partial \eta_a} = \frac{W_a}{W_b}\frac{\partial \psi_b(1,\xi)}{\partial \eta_b} \tag{9}$$

and the dimensionless outlet temperature is

$$\theta_F = 1-\psi_F = \frac{T_F - T_I}{T_s - T_I}. \tag{10}$$

Inspection of the above equations shows that boundary conditions for both channels are conjugated and the analytical solutions to this type of problem may be solved simultaneously by use of the power-series expansion technique.

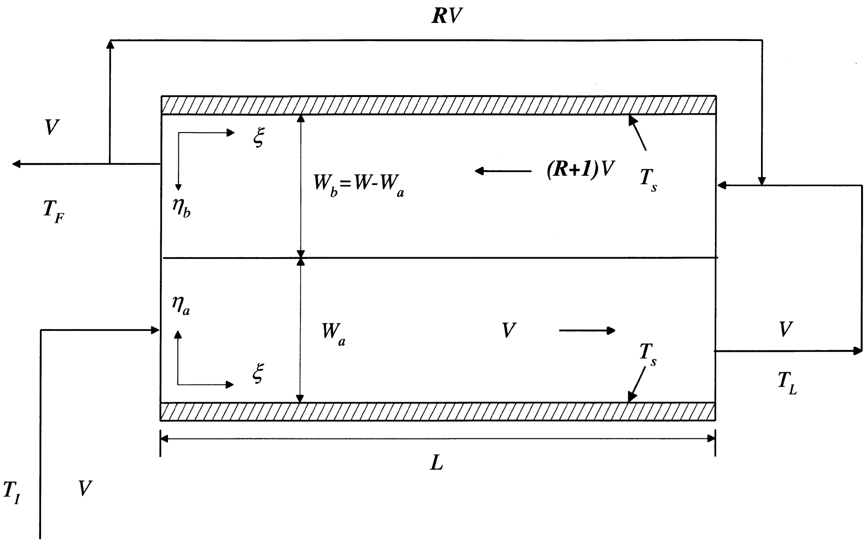

Fig. 1. Double-pass parallel-plate heat exchanger with recycle at the outlet.

Separation of variables in the form

$$\psi_a(\eta_a,\xi)=\sum_{m=1}^{\infty}S_{a,m}F_{a,m}(\eta_a)G_m(\xi) \tag{11}$$

$$\psi_b(\eta_b,\xi)=\sum_{m=1}^{\infty}S_{b,m}F_{b,m}(\eta_b)G_m(\xi) \tag{12}$$

applied to equations (1) and (2) yields

$$G_m(\xi)=e^{-\lambda_m(1-\xi)} \tag{13}$$

$$F''_{a,m}(\eta_a)-\left[\frac{\lambda_m W_a^2 v_a(\eta_a)}{L\alpha}\right]F_{a,m}(\eta_a)=0 \tag{14}$$

$$F''_{b,m}(\eta_b)-\left[\frac{\lambda_m W_b^2 v_b(\eta_b)}{L\alpha}\right]F_{b,m}(\eta_b)=0 \tag{15}$$

and also the boundary conditions in equations (6)–(9) can be rewritten as

$$F_{a,m}(0)=0 \tag{16}$$

$$F_{b,m}(0)=0 \tag{17}$$

$$S_{a,m}F_{a,m}(1)=S_{b,m}F_{b,m}(1) \tag{18}$$

$$-S_{a,m}F'_{a,m}(\eta_{a,m})_{|\eta_a=1}=\frac{W_a}{W_b}S_{b,m}F'_{b,m}(\eta_b)_{|\eta_b=1} \tag{19}$$

where the primes on $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$ denote the differentiations with respect to η_a and η_b, respectively.

Combination of equations (18) and (19) results in

$$\frac{S_{a,m}}{S_{b,m}}=\frac{F_{b,m}(1)}{F_{a,m}(1)}=-\frac{W_a}{W_b}\frac{F'_{b,m}(\eta_b)_{|\eta_b=1}}{F'_{a,m}(\eta_a)_{|\eta_a=1}} \tag{20}$$

in which the eigenfunctions $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$ were assumed to be polynomials to avoid the loss of generality. With the use of equations (16) and (17), we have

$$F_{a,m}(\eta_a)=\sum_{n=0}^{\infty}d_{mn}\eta_a^n,\quad d_{m0}=0,\quad d_{m1}=1(\text{selected}) \tag{21}$$

$$F_{b,m}(\eta_b)=\sum_{n=0}^{\infty}e_{mn}\eta_b^n,\quad e_{m0}=0,\quad e_{m1}=1(\text{selected}). \tag{22}$$

Substituting equations (21) and (22) into equations (14) and (15), all the coefficients $d_{m,n}$ and $e_{m,n}$ may be obtained in terms of eigenvalue λ_m, as referred to in the Appendix. Therefore, it is easy to solve all eigenvalues from equation (20) and the eigenfunctions associated with the corresponding eigenvalues are also well defined by equations (21) and (22). The eigenvalues, λ_m, thus calculated, include a positive set, a negative set and a complex set, the eigenvalues indicated in Table 1 are the set dominant in the system. Equations (14) and (15) are the special cases of Sturm–Liouville problem with the velocity distributions in both channels having opposite signs over the interval in question.

The dimensionless outlet temperature ψ_F which is referred to as the bulk temperature, may be calculated by

$$\psi_F=\frac{-\int_0^1 v_b W_b B\psi_b(\eta_b,0)\,d\eta_b}{(R+1)V}=\frac{-1}{(R+1)G_Z(1-\Delta)}\times\sum_{m=1}^{\infty}\frac{e^{-\lambda_m}S_{b,m}}{\lambda_m}[F'_{b,m}(1)-F'_{b,m}(0)] \tag{23}$$

Table 1
Eigenvalues and expansion coefficients as well as dimensionless outlet temperatures in double-pass devices with recycle for $\Delta = 0.5$ and $R = 1$. $G_Z\lambda_1 = -9.67884$ and $G_Z\lambda_2 = -19.1971$

G_Z	m	λ_m	$S_{a,m}$	$S_{b,m}$	$\psi_F(\lambda_1)$	$\psi_F(\lambda_1, \lambda_2)$
1	1	−9.67884	2.3×10^{-4}	1.4×10^{-5}	0.19527	0.19691
	2	−19.1971	-6.4×10^{-11}	-8.4×10^{-12}		
10	1	−0.96788	2.0	1.2×10^{-1}	0.28491	0.28491
	2	−1.91971	1.3×10^{-7}	-1.7×10^{-8}		
100	1	−0.09679	11.6	7.1×10^{-1}	0.74149	0.74149
	2	−0.19197	7.2×10^{-6}	-9.5×10^{-7}		
1000	1	−0.00968	16.5	1.0	0.96489	0.96489
	2	−0.01920	4.9×10^{-5}	-6.5×10^{-5}		

and may be examined by equations (24) and (26) which is readily obtained from the following overall energy balance on both channels at both ends

$$\psi_F = \frac{-\int_0^1 v_b W_b B\psi_b(\eta_b,1)\,d\eta_b - \int_0^1 v_a W_a B\psi_a(\eta_a,1)\,d\eta_a}{RV}$$

or, by using equations (14), (15) and (19)

$$\psi_F = \frac{1}{RG_Z(1-\Delta)}\left\{\sum_{m=1}^{\infty}\frac{S_{b,m}F'_{b,m}(0)}{\lambda_m(1-\Delta)} + \sum_{m=1}^{\infty}\frac{S_{a,m}F'_{a,m}(0)}{\lambda_m\Delta}\right\} \tag{24}$$

and

$$\begin{aligned} V(1-\psi_F) &= \int_0^1 \frac{\alpha BL}{W_a}\frac{\partial\psi_a(0,\xi)}{\partial\eta_a}\,d\xi + \int_0^1 \frac{\partial\psi_b(0,\xi)}{\partial\eta_b}\,d\xi \\ &= \left[\frac{\alpha BL}{W_a}\sum_{m=1}^{\infty} S_{a,m}F'_{a,m}(0)\int_0^1 e^{-\lambda_m(1-\xi)}\,d\xi + \frac{\alpha BL}{W_b}\sum_{m=1}^{\infty} S_{b,m}F'_{b,m}(0)\int_0^1 e^{-\lambda_m(1-\xi)}\,d\xi\right] \end{aligned} \tag{25}$$

or

$$\psi_F = 1 - \frac{1}{G_Z}\left[\sum_{m=1}^{\infty}\frac{(1-e^{-\lambda_m})}{\lambda_m\Delta}S_{a,m}F'_{a,m}(0) + \sum_{m=1}^{\infty}\frac{(1-e^{-\lambda_m})}{\lambda_m(1-\Delta)}S_{b,m}F'_{b,m}(0)\right]. \tag{26}$$

In equation (24) the term $-\int_0^1 v_b W_b \psi_b(\eta_b,1)\,d\eta_b$, denotes the inlet stream of channel b, and $\int_0^1 v_a W_a \psi_a(\eta_a,1)\,d\eta_a$ and $\psi_F RV$ are the outlet stream of channel a and the recycle stream from channel b, respectively; in equation (25) the left-hand side refers to the net outlet energy, while the right-hand side is the total amount of heat transfer from hot plates to the fluid.

Mathematically, with known Graetz number (G_Z) and reflux ratio (R), as well as the ratio of channel thickness ($\Delta = W_a/W$), the eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_m, \ldots$) can be calculated from equation (20). Fortunately, it is seen from equations (A3) and (A4) in the Appendix that G_Z and λ_m always appear together and one may further define a modified eigenvalue as $\lambda_m^* = G_Z\lambda_m =$ constant. In this case λ_m^* is independent of G_Z and one need not repeat the computation of λ_m for different G_Z. Since both $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$ are not orthogonal functions for the system of interest, the expansion coefficients, $S_{a,m}$ and $S_{b,m}$, cannot be completely found by the orthogonality rule. Here we assume that equations (11) and (12) converge rapidly and only the first two terms in each equation as well as the first two eigenvalues, λ_1 and λ_2, are needed to take. The rapid convergence of equations (11) and (12) will be verified later. Accordingly, we have, from equations (20), (23), (24) and (26)

$$\frac{S_{a,1}}{S_{b,1}} = \frac{F_{b,1}(1)}{F_{a,1}(1)} \tag{27}$$

$$\frac{S_{a,2}}{S_{b,2}} = \frac{F_{b,2}(1)}{F_{a,2}(1)} \tag{28}$$

$$\psi_F \approx \frac{1}{(R+1)G_Z(1-\Delta)}\left\{\frac{e^{-\lambda_1}S_{b,1}}{\lambda_1}[F'_{b,1}(1)-F'_{b,1}(0)] + \frac{e^{-\lambda_2}S_{b,2}}{\lambda_2}[F'_{b,2}(1)-F'_{b,2}(0)]\right\} \tag{29}$$

$$\psi_F \approx \frac{1}{RG_Z(1-\Delta)}\left\{\frac{S_{b,1}F'_{b,1}(0)}{\lambda_1(1-\Delta)} + \frac{S_{b,2}F'_{b,2}(0)}{\lambda_2(1-\Delta)} + \frac{S_{a,1}F'_{b,1}(0)}{\lambda_1\Delta} + \frac{S_{a,2}F'_{b,2}(0)}{\lambda_2\Delta}\right\} \tag{30}$$

$$\psi_F \approx 1 - \frac{1}{G_Z}\left\{\frac{(1-e^{-\lambda_1})}{\lambda_1\Delta}S_{a,1}F'_{a,1}(0) + \frac{(1-e^{-\lambda_2})}{\lambda_2\Delta}S_{a,2}F'_{b,2}(0) + \frac{(1-e^{-\lambda_1})}{\lambda_1(1-\Delta)}S_{b,1}F'_{b,1}(0)\right.$$

$$+\frac{(1-e^{-\lambda_2})}{\lambda_2(1-\Delta)}S_{b,2}F'_{b,2}(0)\Bigg\}. \quad (31)$$

Therefore, with known Δ, R and G_Z, once the first two eigenvalues, λ_1 and λ_2, are calculated from equation (20), the corresponding values of $S_{a,1}$, $S_{a,2}$, $S_{b,1}$, $S_{b,2}$ and ψ_F can be obtained by solving equations (27)–(31) simultaneously. Table 1 shows some results of eigenvalues and their corresponding expansion coefficients for $\Delta = 0.5$, $R = 1$ and $G_Z = 1$, 10, 100 and 1000, while Table 2 shows the corresponding results of dimensionless outlet temperatures. It is seen from Table 2 that the infinite-series equations for temperature distribution are really converged very rapidly even for the very small value of Graetz number such as $G_Z = 1$, and only the first negative eigenvalue is necessary to be considered during the calculation of temperature distribution. Accordingly, the simplified equations, equations (29)–(31), obtained from equations (23), (24) and (26), respectively, are really acceptable.

The dimensionless temperature at the end point ($\xi = 1$) of lower channel can be expressed as follows:

$$\psi_L = \frac{\int_0^1 v_a W_a B \psi_a(\eta_a, 1)\,d\eta_a}{V} = \frac{1}{G_Z\Delta}\sum_{m=1}^{\infty}\frac{S_{a,m}}{\lambda_m}[F'_{a,m}(1)-F'_{a,m}(0)]. \quad (32)$$

3. Temperature distribution in a single-pass device

For the single-pass device of the same size without recycle, $R = 0$, the impermeable plate barrier in Fig. 1 is removed and thus, $\Delta = 1$, $W_a = W$ and $\eta_a = \eta_0$. As we can see, however, it is difficult to obtain the solutions of a single-pass device without recycle from equations (29)–(31) by setting $\Delta = 1$ and $R = 0$. The velocity distribution and equation of energy in dimensionless form may then be written as

$$\frac{\partial^2\psi_0(\eta_0,\xi)}{\partial\eta_0^2} = \left(\frac{W^2 v_0(\eta_0)}{L\alpha}\right)\frac{\partial\psi_0(\eta_0,\xi)}{\partial\xi} \quad (33)$$

$$v_0(\eta_0) = \frac{V}{WB}(6\eta_0 - 6\eta_0^2), \quad 0 \leqslant \eta_0 \leqslant 1 \quad (34)$$

in which

$$\eta_0 = \frac{x}{W}, \quad \xi = \frac{z}{L}, \quad \psi_0 = \frac{T_0 - T_s}{T_I - T_s},$$
$$G_Z = \frac{VW}{\alpha BL}, \quad \theta_0 = 1 - \psi_0 = \frac{T_0 - T_I}{T_s - T_I}. \quad (35)$$

The boundary conditions for solving equation (33) are

$$\psi_0(0,\xi) = 0 \quad (36)$$

$$\psi_0(1,\xi) = 0 \quad (37)$$

Table 2
The improvement of the transfer efficiency with reflux ratio and barrier position as parameters

I_h (%)	$R = 0.5$			$R = 1.0$			$R = 2.0$			$R = 5.0$		
	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$
$G_Z = 1$	−16.65	−22.75	−21.66	−15.08	−19.58	−19.15	−11.16	−15.08	−16.42	−6.22	−8.71	−10.55
10	15.94	1.41	−2.72	20.77	7.73	2.56	27.18	16.66	8.67	35.94	29.41	21.47
100	251.93	121.75	75.12	294.59	160.46	89.31	373.74	229.18	147.28	507.12	372.03	248.27
1000	475.28	178.60	140.21	558.73	246.23	158.69	816.99	384.73	218.48	1503.6	795.38	421.51

$$\psi_0(\eta_0, 0) = 1. \tag{38}$$

Introduce the form of separation of variables for ψ_0 as

$$\psi_0(\eta_0, \xi) = \sum_{m=1}^{\infty} S_{0,m} F_{0,m}(\eta_0) G_{0,m}(\xi). \tag{39}$$

Substitution of equation (39) into equation (33) yields

$$G_{0,m}(\xi) = e^{-\lambda_{0,m}\xi} \tag{40}$$

$$F''_{0,m}(\eta_0) - \left[\frac{\lambda_{0,m} W^2 v_0(\eta_0)}{L\alpha}\right] F_{0,m}(\eta_0) = 0 \tag{41}$$

and the boundary conditions in equations (36) and (37) can be rewritten as

$$F_{0,m}(0) = 0 \tag{42}$$

$$F_{0,m}(1) = 0 \tag{43}$$

in which the eigenfunctions $F_{0,m}(\eta_0)$ were assumed to be polynomials to avoid the loss of generality. With the use of equation (42), we have

$$F_{0,m}(\eta_0) = \sum_{n=0}^{\infty} k_{mn} \eta_0^n, \quad k_{m0} = 0, \quad k_{m1} = 1(\text{selected}). \tag{44}$$

Substituting equation (44) into equation (43), all the coefficients k_{mn} may be obtained in terms of eigenvalue $\lambda_{0,m}$, as referred to in the Appendix. Furthermore, by applying the boundary condition in equation (43) into equation (44), one obtains

$$0 = \sum_{n=0}^{\infty} k_{mn}. \tag{45}$$

Thus, the eigenvalues can be calculated once the Graetz numbers G_Z are specified.

The orthogonality condition of the single pass system is easy to show as

$$\int_0^1 v_0(\eta_0) S_{0,m} S_{0,n} F_{0,m} F_{0,n} \, d\eta_0 = 0, \quad n \neq m. \tag{46}$$

Also, from the inlet condition, equation (38), we have

$$1 = \sum_{m=1}^{\infty} S_{0,m} F_{0,m}(\eta_0). \tag{47}$$

Therefore, with the use of the orthogonality condition, the general expression for the expansion coefficients can be obtained from equation (47) as

$$S_{0,}{}^{m} = \frac{\int_0^1 v_0(\eta_0) F_{0,m}(\eta_0) \, d\eta_0}{\int_0^1 v_0(\eta_0) F_{0,m}^2(\eta_0) \, d\eta_0} = \frac{1}{\lambda_{0,m}} \frac{[F'_{0,m}(1) - F'_{0,m}(0)]}{\left[F'_{0,m}(0) \dfrac{\partial F_{0,m}(0)}{\partial \lambda_{0,m}} - F'_{0,m}(1) \dfrac{\partial F_{0,m}(1)}{\partial \lambda_{0,m}}\right]}. \tag{48}$$

The dimensionless outlet temperature is calculated by

$$\psi_{0,L} = \frac{WB}{V} \int_0^1 v_0(\eta_0) WB \psi_0(\eta_0, 1) \, d\eta_0 = \frac{1}{G_Z} \sum_{m=1}^{\infty} \frac{e^{-\lambda_{0,m}} S_{0,m}}{\lambda_{0,m}} \{F'_{0,m}(1) - F'_{0,m}(0)\} \tag{49}$$

and may be examined by equation (51), which is readily obtained from the following overall energy balance on both channels

$$\begin{aligned} V(1 - \psi_{0,L}) &= \int_0^1 \frac{\alpha BL}{W} \frac{\partial \psi_0(0, \xi)}{\partial \eta_0} d\xi - \int_0^1 \frac{\alpha BL}{W} \frac{\partial \psi_0(1, \xi)}{\partial \eta_0} d\xi \\ &= \frac{\alpha BL}{W} \left[\sum_{m=1}^{\infty} S_{0,m} F'_{0,m}(0) \int_0^1 e^{-\lambda_{0,m}\xi} d\xi - \sum_{m=1}^{\infty} S_{0,m} F'_{0,m}(1) \int_0^1 e^{-\lambda_{0,m}\xi} d\xi\right] \end{aligned} \tag{50}$$

or

$$\psi_{0,L} = 1 - \frac{1}{G_Z} \sum_{m=1}^{\infty} \left[\frac{(1 - e^{-\lambda_{0,m}})}{\lambda_{0,m}} S_{0,m} F'_{0,m}(0) - \frac{(1 - e^{-\lambda_{0,m}})}{\lambda_{0,m}} S_{0,m} F'_{0,m}(1)\right]. \tag{51}$$

The calculation procedure for a single-pass device without recycle is rather simpler than that for a double-pass device with recycle. Mathematically, once the Graetz number is given, the eigenvalues can be calculated from equation (45) and the associated expansion coefficients $S_{0,m}$ are then calculated from equation (48). Fortunately, it is also seen from equation (A6) in the Appendix that G_Z and $\lambda_{0,m}$ always appear together and we may further define a modified eigenvalue as $\lambda^*_{0,m} = G_Z \lambda_{0,m} = \text{constant}$. In this case $\lambda^*_{0,m}$ is independent of G_Z and one need not repeat the computation of $\lambda_{0,m}$ for different G_Z. Finally, the dimensionless outlet temperature $(\theta_{0,L} = 1 - \psi_{0,L})$ may be obtained from either equation (49) or equation (51).

4. Improvement of transfer efficiency

Now, the Nusselt number for a double-pass device with recycle may be defined as

$$\overline{Nu} = \frac{\bar{h} W}{k} \tag{52}$$

in which the average heat transfer coefficient is defined as

$$q = \bar{h}(2BL)(T_s - T_I). \tag{53}$$

Since

$$\bar{h}(2BL)(T_s - T_I) = V\rho c_p(T_F - T_I) \quad (54)$$

or

$$\bar{h} = \frac{V\rho c_p}{2BL}\left(\frac{T_F - T_I}{T_s - T_I}\right) = \frac{V\rho c_p}{2BL}(1-\psi_F) \quad (55)$$

thus

$$\overline{Nu} = \frac{\bar{h}W}{k} = \frac{VW}{2\alpha BL}(1-\psi_F) = 0.5G_Z\theta_F. \quad (56)$$

Similarly, for a single-pass device without recycle

$$\overline{Nu_0} = \frac{\overline{h_0}W}{k} = \frac{VW}{2\alpha BL}(1-\psi_{0,L}) = 0.5G_Z\theta_{0,L}. \quad (57)$$

The improvement of performance by employing a double-pass device with recycle is best illustrated by calculating the percentage increase in heat-transfer rate, based on the heat transfer of a single-pass device with the same device dimension and operating conditions, but without impermeable plate barrier and recycle, as

$$I_h = \frac{\overline{Nu} - \overline{Nu_0}}{\overline{Nu_0}} = \frac{\psi_{0,L} - \psi_F}{1-\psi_{0,L}} = \frac{\theta_F - \theta_{0,L}}{\theta_{0,L}}. \quad (58)$$

5. Increment of power consumption

The friction loss in conduits may be estimated by

$$\ell w_f = \frac{2f\bar{v}^2 L}{D_e} \quad (59)$$

where $\bar{v}$ and D_e denote the bulk velocities in the conduits and the equivalent diameters of the conduits, respectively, while f is the friction factor which is the function of Reynolds number, Re. Accordingly

$$\overline{v_0} = \frac{V}{BW}, \quad \overline{v_a} = \frac{V}{B\Delta W}, \quad \overline{v_b} = \frac{(R+1)V}{B(1-\Delta)W} \quad (60)$$

$$D_{e,0} = 2W, \quad D_{e,a} = 2\Delta W, \quad D_{e,b} = 2(1-\Delta)W. \quad (61)$$

Thus, from equations (60) and (61), one obtains

$$Re_0 = Re_a = \frac{Re_b}{(R+1)} \quad (62)$$

and then we have, for laminar flow

$$f_0 = f_a = f_b(R+1). \quad (63)$$

The increment of power consumption, I_p, may be defined as

$$I_p = \frac{(\ell w_{f,a} + \ell w_{f,b}) - (\ell w_{f,0})}{\ell w_{f,0}}. \quad (64)$$

Substitution of equations (59)–(63) into equation (64) results in

$$I_p = \frac{1}{\Delta^3} + \frac{R+1}{(1-\Delta)^3} - 1. \quad (65)$$

Some results for I_p are presented in Table 3. It is seen from this table that the increment of power consumption does not depend on Graetz number but increases with the reflux ratio or as Δ goes away from 0.5, especially for $\Delta > 0.5$.

6. Results and discussion

6.1. Outlet temperature and transfer efficiency in a double-pass device with recycle

Table 1 shows some calculation results of the first two eigenvalues and their associated expansion coefficients, as well as the dimensionless outlet temperatures, for $\Delta = 0.5$, $R = 1$, and $G_Z = 1$, 10, 100 and 1000. It was observed that due to the rapid convergence, only the first negative eigenvalue is necessary to be considered during the calculation of temperature distribution. Figure 2 shows the relation of another more practical form of dimensionless outlet temperature $\theta_b(\eta_b, 0) = \theta_F$ vs G_Z with the reflux ratio R as a parameter for $\Delta = 0.5$ while Fig. 3 with the ratio of channel thickness Δ as parameter. It is found in Figs 2 and 3 that for a fixed reflux ratio, the dimensionless average outlet temperature decreases with increasing the Graetz number G_Z owing to the short residence time of fluid, but increases as the reflux ratio R increases, due to the preheating effect, or when the ratio of channel thickness Δ decreases, due to the creation of high convective heat-transfer coefficient in channel a where the temperature difference between the heated plate and fluid is larger than in channel b. Figure 4 shows the theoretical average Nusselt numbers $\overline{Nu}$ vs G_Z, with the reflux ratio as a parameter for $\Delta = 0.5$ while Fig. 5 with reflux ratio and the ratio of the channel thickness Δ as parameters. It is seen in Figs 4 and 5 that $\overline{Nu}$ increases with increasing R, but with decreasing Δ, as shown in Fig. 6.

Table 3
The increment of power consumption with reflux ratio and barrier position as parameters

	I_p (%)		
R	$\Delta = 0.25$	$\Delta = 0.5$	$\Delta = 0.75$
0.5	66.56	19.00	97.37
1.0	67.74	23.00	129.37
2.0	70.11	31.00	193.37
5.0	77.22	55.00	385.37

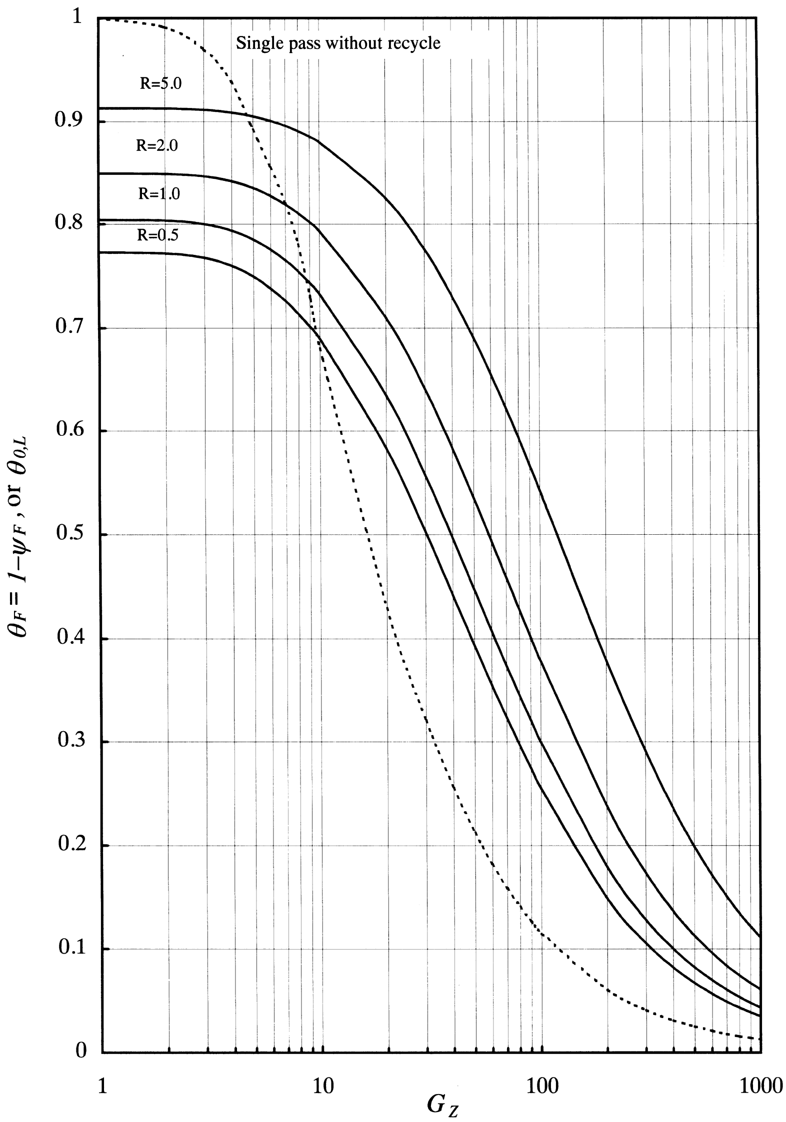

Fig. 2. Dimensionless outlet temperatures of both devices vs G_Z with reflux ratio as parameter; $\Delta = 0.5$.

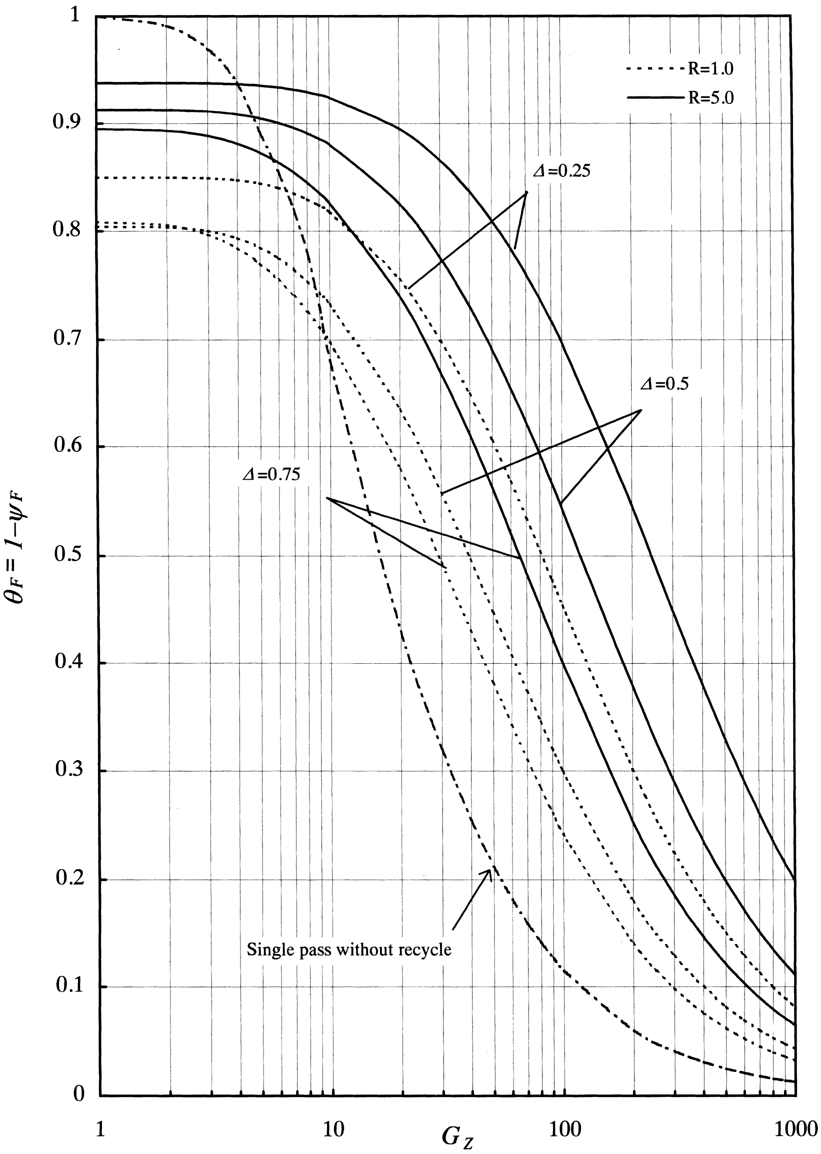

Fig. 3. Dimensionless outlet temperature vs G_Z with Δ as parameter; $R = 1$ and 5.

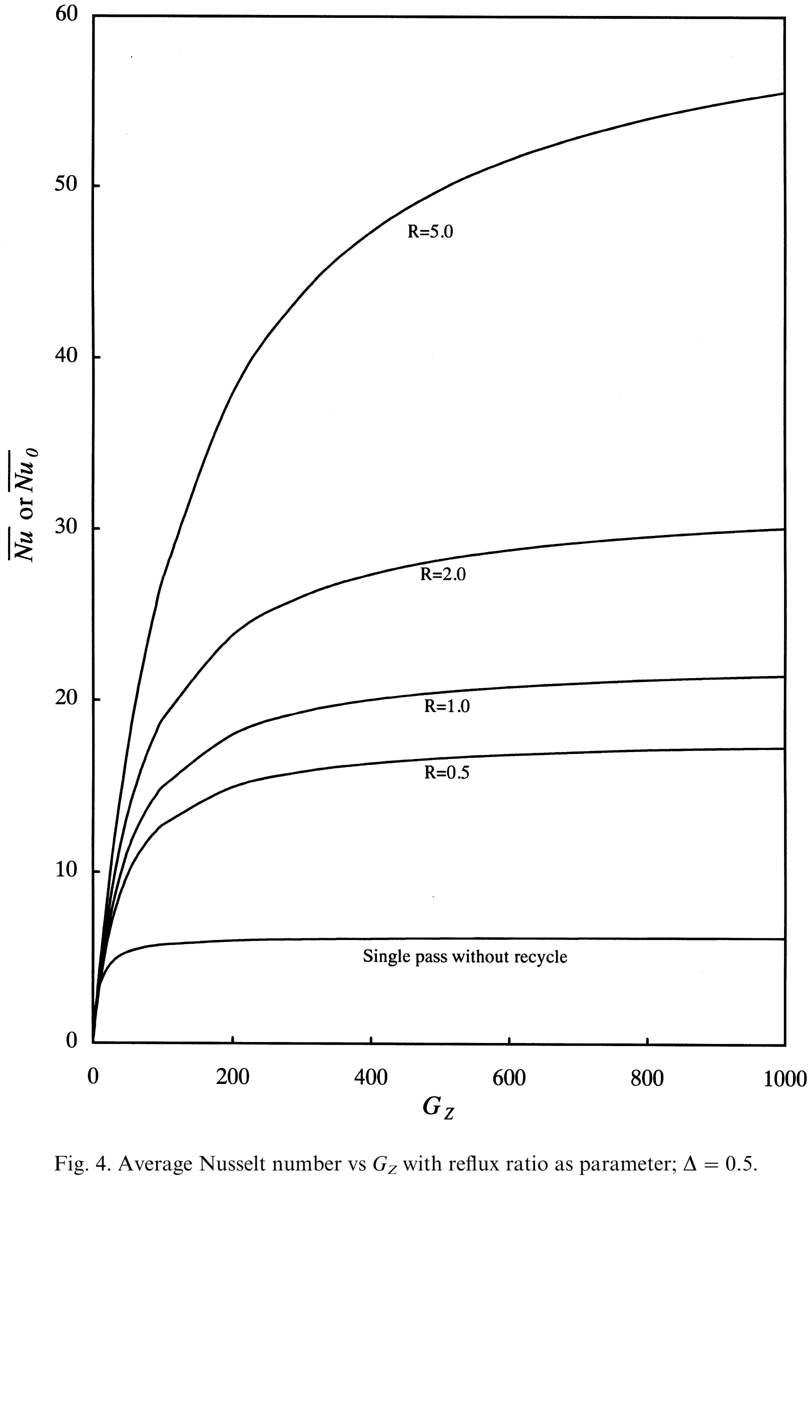

Fig. 4. Average Nusselt number vs G_Z with reflux ratio as parameter; $\Delta = 0.5$.

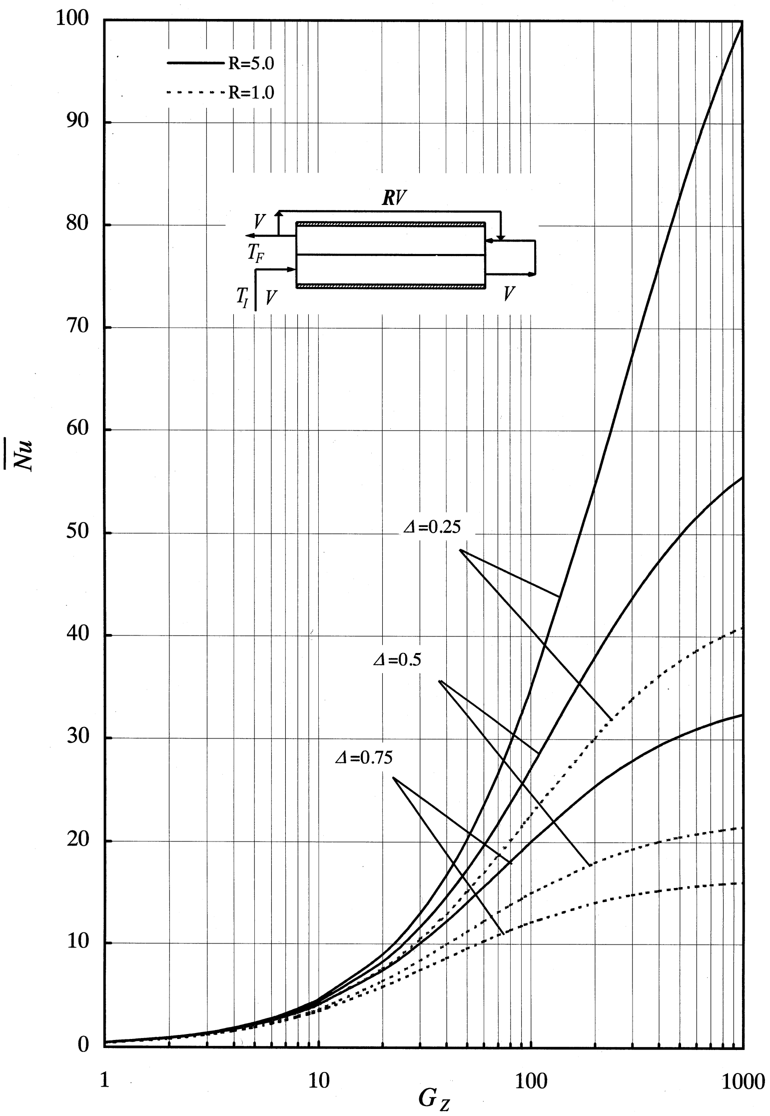

Fig. 5. Average Nusselt number vs G_Z with Δ as parameter; $R = 1$ and 5.

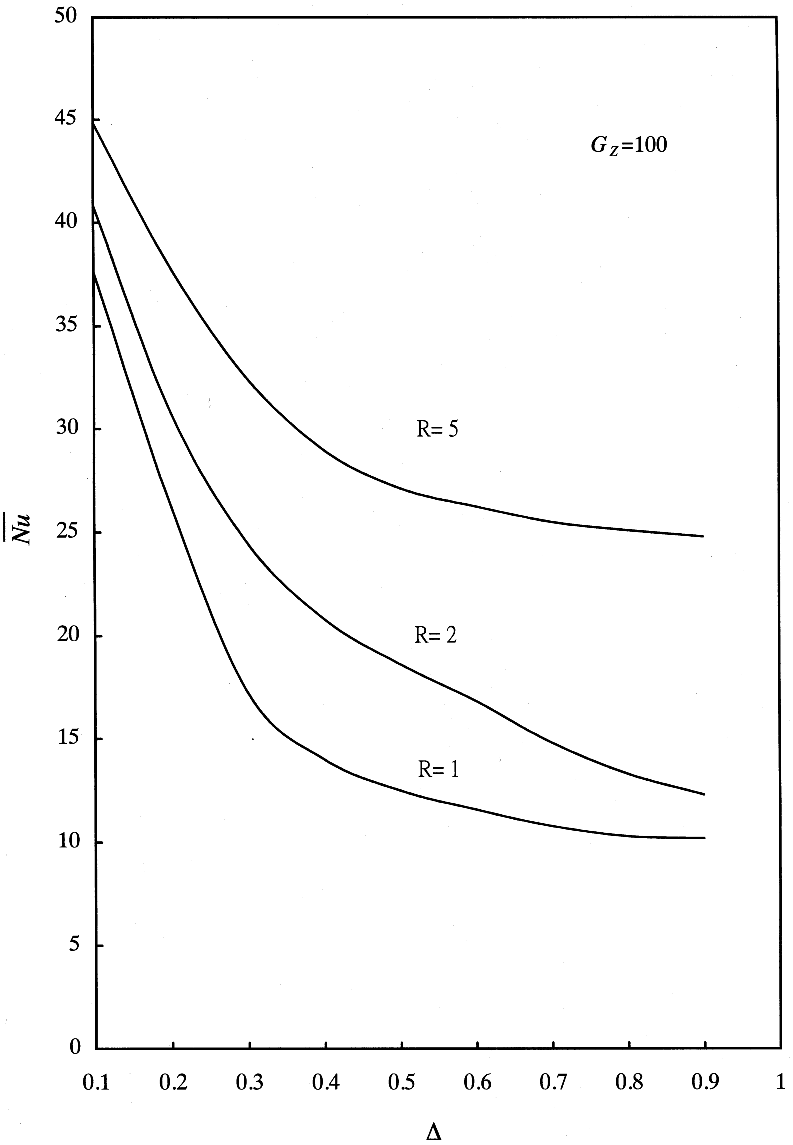

Fig. 6. Average Nusselt number vs Δ with reflux ratio as parameter; $G_Z = 100$.

6.2. Improvement in transfer efficiency based on a single-pass device without recycle

Figures 2 and 3 also show that the theoretical dimensionless outlet temperature $\theta_{0,L}$ in a single-pass device without recycle, decreases with increasing Graetz number G_Z due to the short residence time. Further, the comparison of dimensionless outlet temperatures, $\overline{\theta_F}$ and $\theta_{0,L}$, as well as average Nusselt numbers, $\overline{Nu}$ and $\overline{Nu_0}$, may be observed from Figs 2–4. It is seen from Figs 2 and 3 that the difference $(\theta_F - \theta_{0,L})$ of outlet temperatures increases with G_Z for intermediate G_Z values, but then turns to decreasing and seems to reach zero as G_Z approaches infinite, with any values of R and Δ. On the other hand as shown in Fig. 4, $(\overline{Nu} - \overline{Nu_0})$ increases with G_Z or with reflux ratio R, but the increase with G_Z is limited as G_Z approaches infinite. It is also noted from Figs 2 and 3 that even for very low G_Z, θ_F still can be higher than, or equal to, $\theta_{0,L}$ if F is high enough or if Δ is small enough. In other words, for obtaining the improvement in transfer efficiency under very low Graetz number, higher reflux ratio or lower ratio of channel thickness is needed.

Some numerical values of the improvement in performance I_h were given in Table 2. The minus signs in Table 2 indicate that when $G_Z = 1$, no improvement in transfer efficiency can be achieved as $R \leqslant 5$ and $\Delta \geqslant 0.25$, and in this case, the single-pass device without recycle is preferred to be employed rather than using the double-pass one with recycle operating at such conditions. As mentioned above, however, when G_Z is smaller than 1, the improvement may be still obtainable when the reflux ratio is high enough or if Δ is small enough. This fact may be observed from Figs 2 and 3. In this case, however, the power consumption will be extremely high due to the use of high R or small Δ, as shown in Table 3 and thus, operation under this particular case is not an economical and feasible way.

7. Conclusion

Heat and mass transfer through a double-pass parallel-plate device with recycle has been investigated analytically with ignoring axial conduction or diffusion. For axial fluid conduction to be ignored, the flow Peclet number should be high enough, say $Pe > 50$. Since Graetz number is defined as $G_Z = Pe(W/L)$, for small values of G_Z, in order to neglect axial fluid conduction or diffusion, (W/L) ratio should not exceed some certain values (i.e. $G_Z = 1$, W/L should be less than 0.02). Therefore, for small G_Z number flows, for this assumption to be valid the limiting values of W/L should be taken into consideration. Application of the recycle-effect concept in a parallel-plate heat- or mass-device with the use of double-pass operation, is technically and economically feasible. Moreover, further improvement in transfer efficiency may be obtained if the smaller ratio of channel thickness Δ is selected. The comparison of transfer efficiencies between the device with recycle and the one without recycle is readily observed from Fig. 4 and Table 2. The improvement of performance is really obtained by employing a double-pass device with recycle, instead of using a single-pass device of same size without recycle. As shown in Table 2, the improvement increases with increasing the Graetz number and reflux ratio, as well as with decreasing the ratio of channel thickness.

The equations of heat or mass transfer through a parallel-plate device with external recycle have been derived by using the expansion technique with the eigenfunctions expanding in terms of an extended power series. Although the eigenfunctions are not orthogonal and the expansion coefficients could not be completely found by the orthogonality rule, however, the power-series expressions for temperature distribution converge very rapidly and only the first two eigenvalues as well as their corresponding eigenfunctions are needed. The expansion coefficients of these eigenfunctions were calculated from the energy-balance equations, simultaneously, in fact, the design of a recycle device will produce the desirable preheating (or premixing) effect and the undesirable effect of decreasing residence time. It is concluded that recycle effect can enhance heat and mass transfer, especially for the case that Graetz number is not sufficiently small enough (say $G_Z \leqslant 1$), and the enhancement increases with the reflux ratio increasing, or the channel thickness decreasing. At very low Graetz number (either small input volume flow rate V or large conduit length L), the residence time is essentially long and should be kept for good performance of open duct (without the plate barrier and recycle). In this case, therefore, the preheating effect by increasing the reflux ratio cannot compensate for the decrease of residence time. On the other hand, the introduction of reflux has a positive effect on the heat and mass transfer for the case of not very small Graetz number. This is due to the preheating effect having more influence than the residence-time effect here.

The present paper is actually the extension of another recycle problem in previous works [32, 33], in which the recycle is introduced from the outlet of channel b to mix with the inlet of channel a, while the heated fluid is withdrawn from the end of channel a. Figure 7 also illustrates some results obtained in ref. [32] with the same parameter values used in Fig. 5 for comparison. With this comparison, the advantage of the present results is evident.

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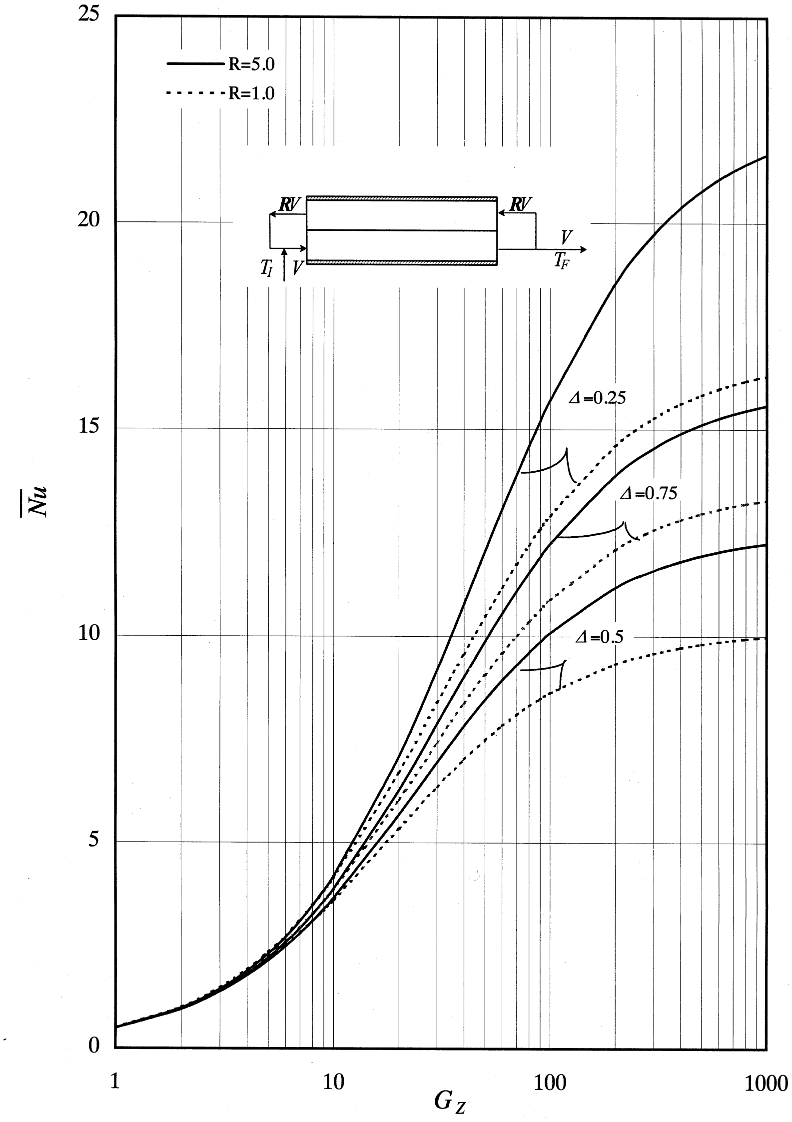

Fig. 7. The results obtained in ref. [33] with the same parameter values used in Fig. 5.

Appendix

Equations (14) and (15) can be rewritten as

$$F''_{a,m}(\eta_a) - \lambda_m G_Z \Delta(6\eta_a - 6\eta_a^2) F_{a,m}(\eta_a) = 0 \quad (A1)$$

$$F''_{b,m}(\eta_b) + \lambda_m G_Z (R+1)(1-\Delta)(6\eta_b - 6\eta_b^2) F_{b,m}(\eta_b) = 0. \quad (A2)$$

Combining equations (A1), (A2), (16), (17), (21) and (22) yields

$$d_{m1} = 1$$
$$d_{m2} = 0$$
$$d_{m3} = 0$$
$$d_{m4} = \frac{1}{2}\lambda_m G_Z \Delta$$
$$\vdots$$
$$d_{mn} = \frac{6}{n(n-1)}\lambda_m G_Z \Delta (d_{m(n-3)} - d_{m(n-4)}) \quad (A3)$$
$$e_{m1} = 1$$
$$e_{m2} = 0$$
$$e_{m3} = 0$$
$$e_{m4} = -\frac{1}{2}\lambda_m G_Z (R+1)(1-\Delta)$$
$$\vdots$$
$$e_{mn} = -\frac{6}{n(n-1)}\lambda_m G_Z (R+1)(1-\Delta)(e_{m(n-3)} - e_{m(n-4)}). \quad (A4)$$

(ii) Equation (41) can be rewritten as

$$F''_{0,m}(\eta_0) - \lambda_{0,m} G_Z (6\eta_0 - 6\eta_0^2) F_{0,m}(\eta_0) = 0. \quad (A5)$$

Substituting equation (44) into equation (A5) with the use of equation (42) yields

$$k_{m1} = 1$$
$$k_{m2} = 0$$
$$k_{m3} = 0$$
$$k_{m4} = \frac{1}{2}\lambda_{0,m} G_Z$$
$$\vdots$$
$$k_{mn} = \frac{6\lambda_{0,m} G_Z}{(n-1)}(k_{m(n-3)} - k_{m(n-4)}). \quad (A6)$$

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